Sains Malaysiana 40(12)(2011): 1429–1435

Thermal Dispersion Effect on Fully Developed Free Convection of Nanofluids in a Vertical Channel

(Kesan Serakan Terma Nanobendalir Olakan Bebas Terbentuk Sepenuhnya dalam Saluran Mengufuk)

TEODOR GROSAN*

ABSTRACT

The effect of thermal dispersion on the steady free convection flow of a nanofluid in a vertical channel is investigated numerically using a single phase model. Considering the laminar and fully developed flow regime a simplified mathematical model is obtained. In the particular cases when solid phase and thermal dispersion effects are neglected the problem was solved analytically. The numerical solution is shown to be in excellent agreement with the close form analytical solution. Nusselt number enhancement with the Grashof number, volume fraction and thermal diffusivity constant increasing has been found.

Keywords: Free convection; fully developed; nanofluid; thermal dispersion

ABSTRAK

Kesan serakan terma pada aliran olakan bebas mantap dalam saluran mengufuk dikaji secara bernombor menggunakan model fasa tunggal. Dengan mengandaikan satu laminar dan regim aliran terbentuk bebas, satu model matematik ringkas telah diperoleh. Dalam kes-kes apabila fasa pepejal dan kesan serakan terma diabaikan, masalah ini dapat diselesaikan secara analitik. Penyelesaian bernombor dan penyelesian analitik didapati bersetuju dengan baik. Peningkatan nombor Nusselt dengan nombor Grashof, pecahan isipadu dan peningkatan pemalar keresapan terma telah diperoleh.

Kata kunci: Nanobendalir; olakan bebas; serakan terma; terbentuk sepenuhnya

INTRODUCTION

Heat transfer in channels occurs in many industrial processes and natural phenomena. It has been, therefore, the subject of many detailed, mostly numerical studies for different flow configurations. Most of the interest in this subject is due to its practical applications, for example, in the design of cooling systems for electronic devices and in the field of solar energy collection. Some of the published papers, such as by Aung (1972), Aung et al. (1972), Barletta (1999), Kumar et al. (2010) Vajravelu and Sastri (1977), are concerned with the evaluation of the temperature and velocity profiles for the vertical parallel-flow fully developed regime.

Enhancement of heat transfer is essential in improving performances and compactness of electronic devices. Usual cooling agents (water, oil, etc.) have relatively small thermal conductivities and therefore heat transfer is not very efficient. Thus, to augment thermal characteristics very small size particles (nanoparticles) where suspended in fluids forming the so call nanofluids. These suspensions of nanoparticles in fluids have physical and chemical properties depending on the concentration and the shape of particles. It was found that a small fraction of nanoparticles added in a base fluid leads to a large increase of the fluid thermal conductivity. A very good description and classification of the nanofluids physical properties can be found in papers such as Daungthongsuk and Wongwises (2007), Wang and Mujumdar (2008) and Kumar et al. (2010).

During the last years several papers studied with nanofluid flow and heat transfer in cavities using the single phase model. Maïga et al.(2004) studied the nanofluid behaviour in a uniformly heated tube, Jou and Tzeng (2006) considered in their research the differentially heated rectangular two dimensional enclosure with different aspect ratio, Tiwari and Das (2007) studied the differentially heated two-sided lid-driven square cavity, Abu-Nada (2008) investigated the flow and heat transfer of a nanofluid over a backward facing step and Oztop and Abu-Nada (2008) carried out nanofluid heat transfer and fluid flow in a partially heated enclosure with different aspect ratio. Very recent papers by Bachok et al. (2010a), Bachok et al. (2010b), Yacob et al. (2011a) and Yacob et al. (2011b) study the fluid flow and heat transfer from different surfaces using the boundary layer approximation. In all these paper the enhancement of heat transfer due to nanofluids special properties was reported.

The chaotic movement of the nanoparticles and sleeping between the fine particles and fluid generate the thermal dispersion effect and this leads to an increase in the energy exchange rates in fluid, see Xuan and Roetzel (2000). Thermal dispersion effects in nanofluids flow in enclosure using a single phase model were analyzed by Khanafer et al. (2003) and Kumar et al. (2010) for a differentially heated rectangular cavity, Khaled and Vafai (2005) studied the heat transfer enhancement through control of thermal dispersion effects in a horizontal channel, while Mokmeli and Saffar-Avval (2010) numerically studied nanofluid heat transfer in a straight tube.

In the present paper, the effect of the thermal dispersion on the steady free convection flow in a long vertical channel using the fully developed flow assumptions was investigated using the single phase thermal dispersion model similar with that proposed by Khanafer et al. (2003).

MATHEMATICAL MODEL

Consider an incompressible nanofluid, which steadily flows between two infinite vertical and parallel plane walls maintained at different constant temperatures extending in the x and y directions, the geometry of the problem, the boundary conditions, and the coordinate system being shown in Figure 1. The fluid rises in the duct driven by buoyancy forces hence, the flow is due only to the difference in temperature gradient. The flow being fully developed the following relations apply here $v = 0$, $\partial v/\partial y = 0$, where v is the velocity in the transversal direction. Thus, from the continuity equation, we get $\partial u/\partial x = 0$ so that $u = u(y)$. Based on the fact that the flow is fully developed we can assume that $T = T(y)$.

The physical properties of the nanofluid are considered constant except for density, which is given by the Boussinesq approximation. We use in this study the heat capacity and the thermal expansion coefficient given in Khanafer et al. (2003):

$$(\rho c_p)_{nf} = (1 - \phi)(\rho c_p)f + \phi(\rho c_p)_s. \tag{1}$$

$$(\rho\beta)_{nf} = (1 - \phi)\, \rho_f \beta_f + \phi \rho_s \beta_s, \tag{2}$$

while for the effective viscosity we consider the model proposed by Brinkman (1952) and valid for high volume fraction (ϕ>0.05):

$$\mu_{nf} = \frac{\mu_f}{(1-\phi)^{2.5}}. \tag{3}$$

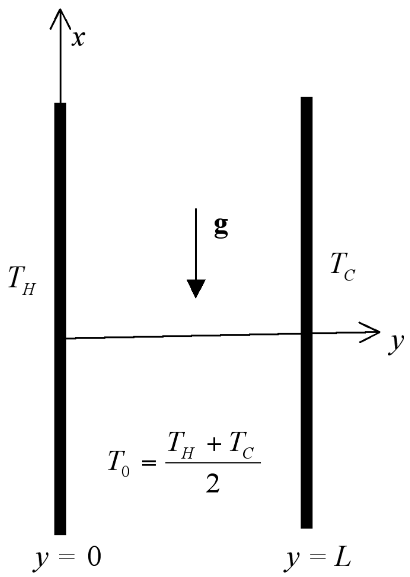


FIGURE 1. Geometry of the problem and co-ordinate system

The effective stagnant thermal conductivity is approximated by the Maxwell-Garnetts model which applies for spherical type particles:

$$\frac{k_{nf}}{k_f} = \frac{k_s + 2k_f - 2\phi\left(k_f - k_s\right)}{k_s + 2k_f + \phi\left(k_f - k_s\right)}. \tag{4}$$

The effective thermal conductivity includes also the thermal dispersion enhancement:

$$k_{eff} = k_{nf} + kd, \tag{5}$$

where the term due to thermal dispersion, k_d, is given by Khaled and Vafai (2005):

$$k_d = C(\rho c_p)_{nf} |u|\, \phi\, L. \tag{6}$$

In (6) C is a constant depending on the diameter of the nanoparticle and its surface geometry.

We limit in this paper to water based nanofluids containing Cu, Al_2O_3, TiO_2 nanoparticles. Nanofluids thermo physical properties are shown in Table 1, see Oztop and Abu-Nada (2008):

TABLE 1. Physical properties of fluid and nanoparticles

Property	Water	Cu	Al_2O_3	TiO_2
c_p (J/kg K)	4179	385	765	686.2
ρ (kg/m^3)	997.1	8933	3970	4250
k (W/m K)	0.613	400	40	8.9538
$\alpha \times 10^7$ (m^2/s)	1.47	1163.1	131.7	3.07
$\beta \times 10^{-5}$ (1/K)	21	1.67	0.85	0.9

Under these assumptions the momentum and energy equations for the flow and heat transfer have the following form:

$$\mu_{nf}\frac{d^2u}{dy^2}+(\rho\beta)_{nf}\,g\left(T-T_0\right)=0, \tag{7}$$

$$\frac{d}{dy}\left(k_{eff}\frac{dT}{dy}\right)=0, \tag{8}$$

subject to the boundary conditions:

$$u(0) = 0,\ u(L) = 0,\ T(0) = T_H,\ T(L) = T_C, \tag{9}$$

where g is the gravitational acceleration.

In order to solve (2) and (3), we introduce the following non-dimensional variables used also by Kumar et al. (2010):

$$U=\frac{u}{U_c},\ Y=\frac{y}{L},\ \theta=\frac{T-T_0}{T_H-T_0}, \tag{10}$$

where U_c and T_0 are the characteristic velocity and temperature given by:

$$U_c=\frac{g\beta_f\left(T_H-T_0\right)L^2}{\nu_f},\ \ T_0=\frac{T_H+T_C}{2}. \tag{11}$$

Using (10) in Eqs. (7)-(9) we obtain the dimensionless governing equations:

$$\frac{d^2U}{dY^2}+\lambda_\phi\theta=0, \tag{12}$$

$$\frac{d}{dY}\left[\left(k_\phi+C\phi\,\mathrm{Pr}\,Gr\sqrt{U^2}\right)\frac{d\theta}{dY}\right]=0, \tag{13}$$

$$U(0) = 0,\ U(1) = 0,\ \theta(1) = -1, \tag{14}$$

where

$$\lambda_\phi=\left(1-\phi\right)^{2.5}\left[\phi\frac{\rho_s}{\rho_f}\frac{\beta_s}{\beta_f}+\left(1-\phi\right)\right],\ k_\phi=\frac{k_{nf}/k_f}{\left(1-\phi\right)+\phi\dfrac{\left(\rho c_p\right)_s}{\left(\rho c_p\right)_f}}, \tag{15}$$

are constants depending on the nanofluid properties and $\mathrm{Pr} = \nu_f/\alpha_f$ and $Gr = g\beta_f(T_H - T_0)L^3/\nu_f^2$ are Prandtl number and Grashof number, respectively.

The physical quantity of interest in this problem is the Nusselt number which for the heated wall is defined as:

$$Nu=\left.\frac{hL}{k_f}\right|_{y=0}, \tag{16}$$

where the convective heat transfer coefficient, h, is given by:

$$-k_{df}\left.\frac{dT}{dY}\right|_{y=0}=h\left(T_H-T_C\right). \tag{17}$$

Using (17) in (16) the dimensionless form of the Nusselt number becomes:

$$Nu=-\left.\frac{k_{nf}}{k_f}\frac{d\theta}{dY}\right|_{Y=0}. \tag{18}$$

Results and Discussions

In the case when thermal dispersion effect is neglected (i.e. $C = 0$) the problem have an analytical solution:

$$\theta(Y) = -2Y + 1. \tag{19}$$

$$U\left(Y\right)=\lambda_\phi\left(\frac{Y^3}{3}-\frac{Y^2}{2}+\frac{Y}{6}\right). \tag{20}$$

Therefore, in this particular case, the Nusselt number, $N = 2k_{nf}/k_f$ depends only by the thermal characteristics of the nanofluid. In addition, in the case of a clear fluid (the nanoparticle phase $\phi = 0$, i.e. $\lambda_\phi = 1$) we recover the classical solution for the natural convection in a vertical channel given by Aung (1972).

Equation (12)-(14) were solved numerically using finite difference discretization for different volume fractions of nanoparticles ϕ = 0, 0.0, 0.1 and 0.2 and different kind of nanoparticles (Cu, Al_2O_3 and TiO_2). In this study we considered constant Prandtl number, Pr = 7, Gr = 10, 100 and 1000 and following Khaled and Vafai (2005) the values for constant C were taken C = 0, 0.1, 0.2, 0.3 and 0.4.

Table 2 to 4 show the Nusselt number for different nanoparticles types (Cu, Al_2O_3 and TiO_2) and different values of the above parameters. In the case C = 0 Nusselt number depends only by thermal properties of the nanofluid and from Table 1 the following relation can be obtained: $Nu_{TiO_2} < NuAl_2O_3 < Nu_{Cu}$. This relation keeps its validity also for $C \neq 0$ and for all considered values of Gr and ϕ. For each type of nanaoparticles, Nusselt number augmentation with the increasing of Gr, ϕ and C is noticed. The above described behaviour of the Nusselt number is also illustrated in Figure 2 to 4. It is seen from Figures 2 to 4 that the dispersion effect is more important for large values of Grashof number Gr and volume fraction ϕ. For example, when a volume fraction of Cu nanoparticles at ϕ = 0.2 is considered and dispersion effect is taken into account (C = 0.4), Nusselt number increases with 1.70% for Gr = 10, 16.69% for Gr = 100 and 142.73% for Gr = 1000.

Figure 5 presents dimensionless velocity and temperature profiles for the thermal diffusivity constant C = 0.4 and different values of volume fraction ϕ when Cu nanoparticles are considered. Figure 5 presents also the analytical solutions for the particular case ϕ = 0 and it is worth to mention that the agreement between the analytical and numerical solution is very good. Reduction of maximum and minimum values of the velocity profiles with the increasing of the volume fraction ϕ is observed while temperature profiles exhibit an oscillatory behavior around the analytical profile.

TABLE 2. Values of Nusselt number for $Gr = 0$, different volume fractions ϕ and different values of constant C

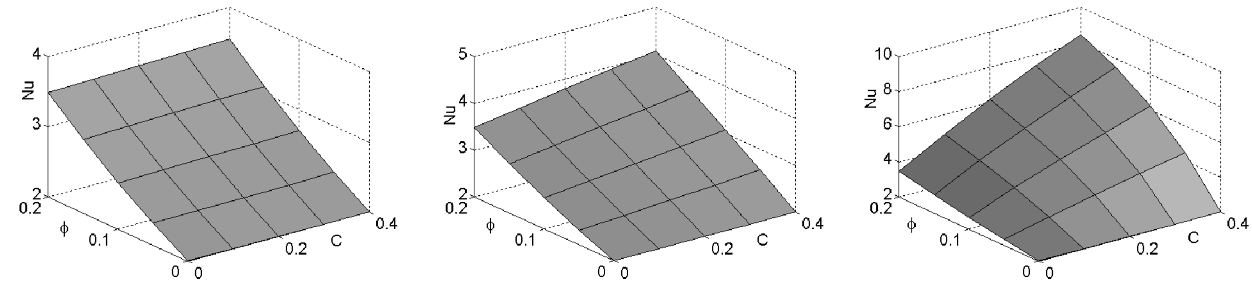
Nanoparticles	C	ϕ 0.05	0.1	0.2
Cu	0	2.314266	2.663273	3.491415
	0.1	2.320430	2.673790	3.506350
	0.2	2.326589	2.684295	3.521265
	0.3	2.332744	2.694787	3.536160
	0.4	2.338893	2.705266	3.551037
Al_2O_3	0	2.300996	2.633786	3.417282
	0.1	2.306959	2.643610	3.430207
	0.2	2.312917	2.653422	3.443117
	0.3	2.318870	2.663222	3.456011
	0.4	2.324819	2.673012	3.468891
TiO_2	0	2.256304	2.535483	3.175914
	0.1	2.262265	2.545300	3.188825
	0.2	2.268221	2.555105	3.201721
	0.3	2.274172	2.564899	3.214600
	0.4	2.280118	2.574681	3.227463

TABLE 3. Values of Nusselt number for $Gr = 100$, different volume fractions ϕ and different values of constant C

Nanoparticles	C	ϕ 0.05	0.1	0.2
Cu	0.1	2.375684	2.767879	3.639889
	0.2	2.436610	2.871254	3.786475
	0.3	2.497056	2.973445	3.931252
	0.4	2.557035	3.074499	4.074298
Al_2O_3	0.1	2.360412	2.731518	3.545861
	0.2	2.419365	2.828161	3.672987
	0.3	2.477865	2.923756	3.798717
	0.4	2.535925	3.018341	3.923101
TiO_2	0.1	2.315696	2.633132	3.304310
	0.2	2.374615	2.729653	3.431152
	0.3	2.433073	2.825089	3.556502
	0.4	2.491083	2.919482	3.680420

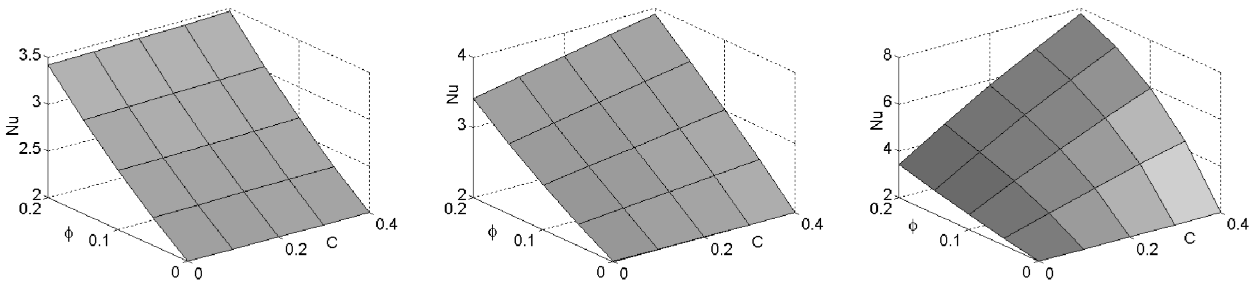
TABLE 4. Values of Nusselt number for $Gr = 1000$, different volume fractions ϕ and different values of constant C

Nanoparticles	C	0.05	ϕ 0.1	0.2
Cu	0.1	2.904678	3.653571	4.891759
	0.2	3.462274	4.567306	6.176981
	0.3	3.986314	5.412138	7.361115
	0.4	4.485737	6.208512	8.474784
Al_2O_3	0.1	2.875651	3.566648	4.643861
	0.2	3.413357	4.420366	5.765317
	0.3	3.922044	5.216572	6.810626
	0.4	4.407197	5.968326	7.797187
TiO_2	0.1	2.830370	3.466077	4.396923
	0.2	3.366958	4.315556	5.507727
	0.3	3.874222	5.106594	6.540027
	0.4	4.357766	5.852679	7.512311



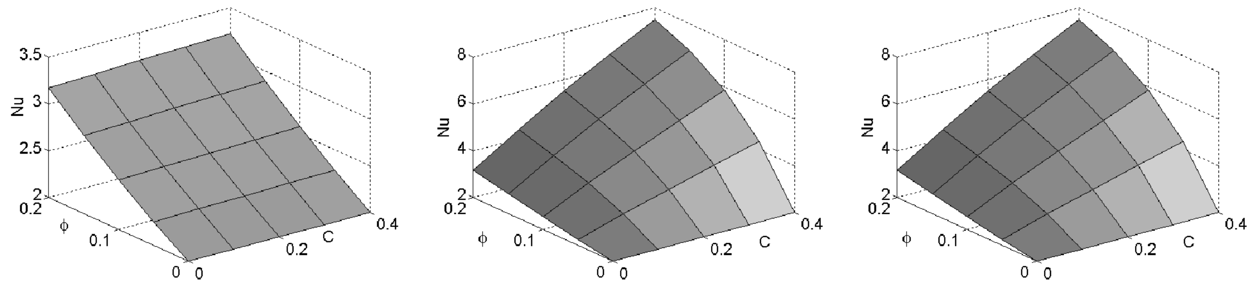
(a) (b) (c)

FIGURE 2. Variation of Nusselt number *Nu* in respect with *C* and ϕ for Cu nanoparticles when a) $Gr = 0$, b) $Gr = 100$ and c) $Gr = 1000$



(a) (b) (c)

FIGURE 3. Variation of Nusselt number *Nu* in respect with *C* and ϕ for Al_2O_3 nanoparticles when a) $Gr = 10$, b) $Gr = 100$ and c) $Gr = 1000$



(a) (b) (c)

FIGURE 4. Variation of Nusselt number *Nu* in respect with *C* and ϕ for TiO2 nanoparticles when a) $Gr = 0$, b) $Gr = 100$ and c) $Gr = 1000$

Figure 6 shows dimensionless velocity and temperature profiles for volume fraction $\phi = 0.2$ and different values of the thermal diffusivity constant *C* when Cu nanoparticles are considered. This figure presents also the analytical solutions for the particular case $C = 0$ and the agreement between the analytical and numerical solution is again very good. Dimensionless velocity and temperature profiles thickness increase with the increasing of the thermal dispersion constant *C*.

The effect of nanoparticles thermal properties on the velocity and temperature profiles are presented in Figure 7. The profiles thickness decreases when Cu, Al_2O_3 and TiO_2 nanoparticles are considered.

CONCLUSION

The effect of thermal dispersion for different nanofluids (Cu, Al_2O_3, TiO_2) was investigated numerically on fully free convection in a differentially heated vertical channel. Heat transfer enhancement depends on Grashof number, type of nanofluid, values of the volume fraction of nanoparticles and thermal dispersion coefficient. Large

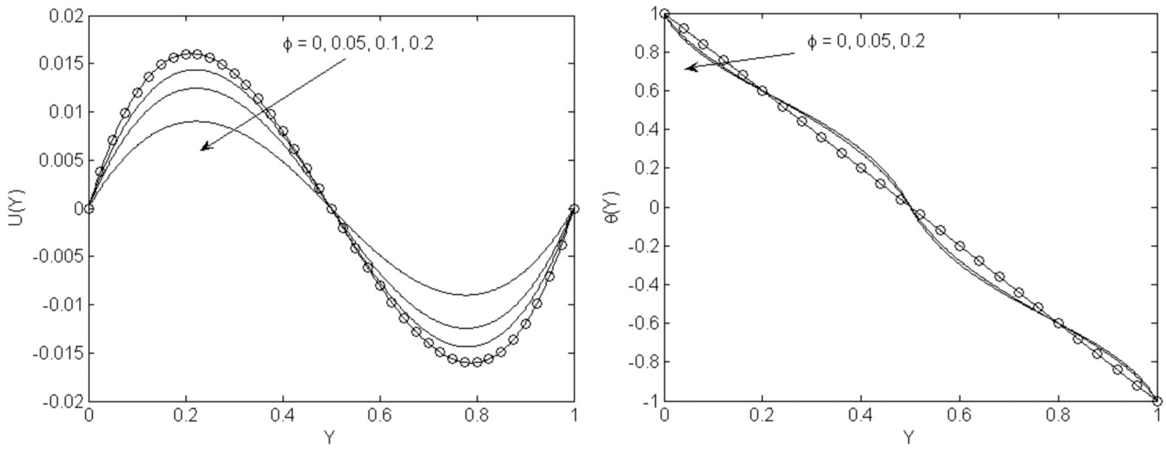


FIGURE 5. Dimensionless velocity and temperature profiles for Cu nanoparticles, $C = 0.4$ and different values of C ('o o o' –analytical solution for $C = 0$)

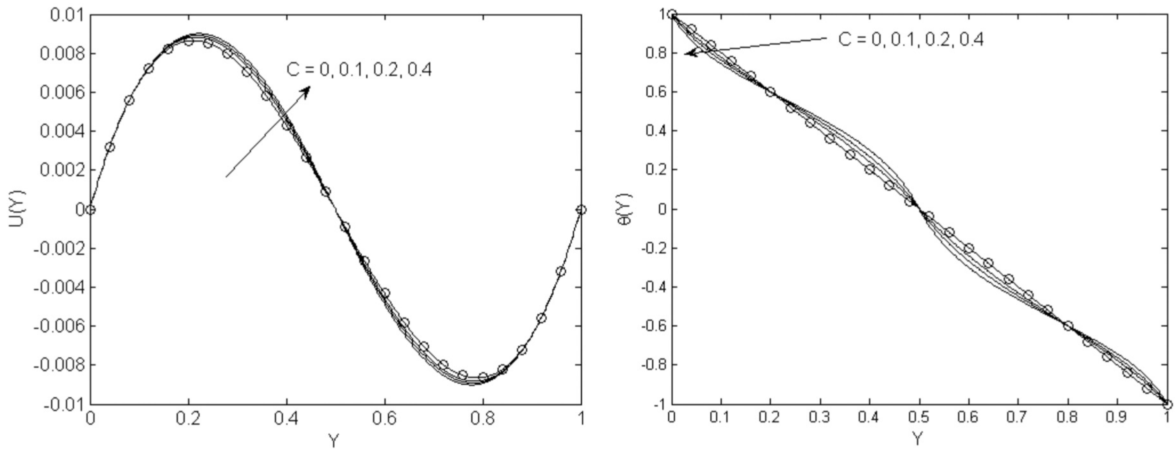


FIGURE 6. Dimensionless velocity and temperature profiles for Cu nanoparticles, $\phi = 0.2$ and different values of C ('o o o' –analytical solution for $C = 0$)

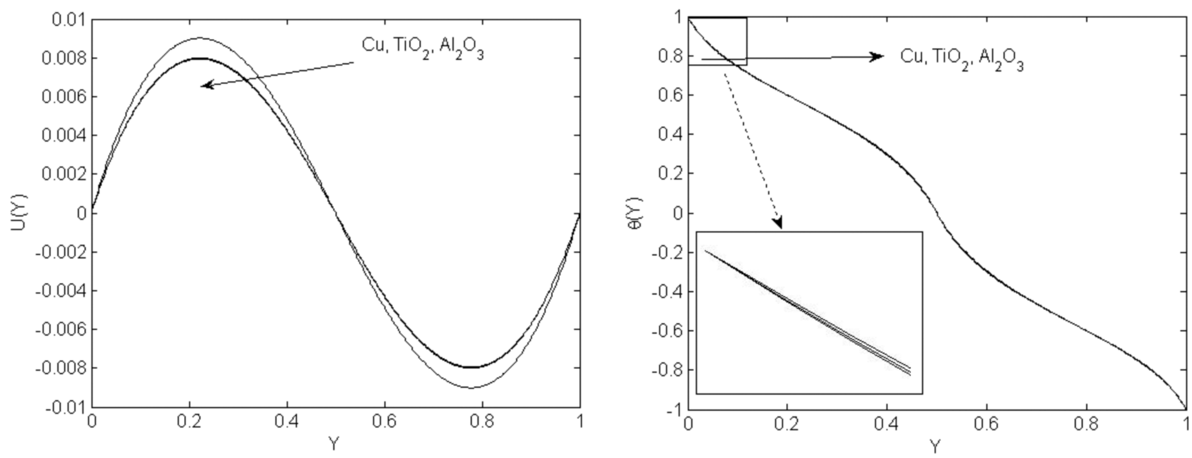


FIGURE 7. Dimensionless velocity and temperature profiles for $\phi = 0.2$, C = 0.4 and different nanoparticles

Nusselt numbers are obtained for large values of *Gr*, ϕ and *C* and for Cu nanoparticles. Thermal dispersion effect considerably increases heat transfer. Further experimental investigation could be necessary for more accurate values of thermal dispersion constant *C*.

ACKNOWLEDGEMENTS

The author would like to express his thanks to the reviewer for his valuable comments and suggestions. This paper was partilly supported from the grant PN-II-RU-TE-2011-3-0013 U-FIScDI, Romania.

REFERENCES

Abu-Nada, E. 2008. Application of nanofluids for heat transfer enhancement of separated flows encountered in a backward facing step. *International Journal of Heat and Fluid Flow* 29: 242-249.

Aung, W. 1972. Fully developed laminar free convection between vertical plates heated asymmetrically. *International Journal of Heat and Mass Transfer* 15: 1577-1580.

Aung, W., Fletcher, L.S. & Sernas, V. 1972. Developing laminar free convection between vertical flat plates with asymmetric heating. *International Journal of Heat and Mass Transfer* 15: 2293-2308.

Bachok N., Ishak, A. & Pop, I. 2010. Boundary-layer flow of nanofluids over a moving surface in a flowing fluid. *International Journal of Thermal Sciences* 49: 1663-1668.

Bachok, N., Ishak, A., Nazar R. & Pop I. 2010. Flow and heat transfer at a general three-dimensional stagnation point in a nanofluid. *Physica B* 405: 4914-4918.

Barletta, A. 1999. Analysis of combined forced and free flow in a vertical channel with viscous dissipation and isothermal-isoflux boundary conditions. *Journal of Heat Transfer* 121: 349-356.

Brinkman, H.C.1952. The viscosity of concentrated suspensions and solutions. *Journal of Chemical Physics* 20: 571-581.

Daungthongsuk, W. & Wongwises, S. 2007. A critical review of convective heat ransfer of nanofluids. *Renewable and Sustainable Energy Reviews* 11: 797-817.

Jou, R.-Y. & Tzeng, S.-C. 2006. Numerical research of nature convective heat transfer enhancement filled with nanofluids in rectangular enclosures. *International Communications in Heat and Mass Transfer* 33: 727-736.

Khaled, A.-R.A. & Vafai, K. 2005. Heat transfer enhancement through control of thermal dispersion effects. *International Journal of Heat and Mass Transfer* 48: 2172-2185.

Khanafer, K., Vafai, K. & Lightstone, M. 2003. Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids. *International Journal of Heat and Mass Transfer* 46: 3639-3653.

Kumar, J.P., Umavathi, J.C., Chamkha, A.J. & Pop, I. 2010. Fully-developed free-convective flow of micropolar and viscous fluids in a vertical channel. *Applied Mathematical Modelling* 34: 1175-1186.

Kumar, S., Prasad, S.K. & Banerjee, J. 2010. Analysis of flow and thermal field in nanofluid using a single phase thermal dispersion model. *Applied Mathematical Modelling* 34: 573-592.

Maïga, S.E.B., Nguyen, C.T., Galanis, N. & Roy, G. 2004. Heat transfer behaviour of nanofluids in a uniformly heated tube. *Superlattices and Microstructures* 35: 543-557.

Mokmeli, A. & Saffar-Avval, M. 2010. Prediction of nanofluid convective heat transfer using the dispersion model. *Int. J. Thermal Sci*. 49: 471-478.

Oztop, H.F. & Abu-Nada, E. 2008. Numerical study of natural convection in partially heated rectangular enclosures filled with nanofluids. *International Journal of Heat and Fluid Flow* 29: 1326-1336.

Tiwari, R.K. & Das, M.K. 2007. Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids. *International Journal of Heat and Mass Transfer* 50: 2002-2018.

Vajravelu, K. & Sastri, K.S. 1977. Fully developed laminar free convection flow between two parallel vertical walls-I. *International Journal of Heat and Mass Transfer* 20: 655-660.

Wang, X.-Q. & Mujumdar, A.S. 2008. A review on nanofluids-part I: Theoretical and numerical investigations. *Brazilian Journal of Chemical Engineering* 25: 613-630.

Xuan, Y. & Roetzel, W. 2000. Conceptions for heat transfer correlation of nanofluids, *International Journal of Heat and Mass Transfer* 43: 3701-3707.

Yacob, N.A., Ishak, A. & Pop, I. 2011. Falkner-Skan problem for a static or moving wedge in nanofluids. *International Journal of Thermal Sciences* 50: 133-139.

Yacob, N.A., Ishak, A., Nazar R. & Pop, I. 2011. Falkner-Skan problem for a static and moving wedge with prescribed surface heat flux in a nanofluid. *International Communications in Heat and Mass Transfer* doi: 10.1016/j.icheatmasstransfer.2010.12.03.

Faculty of Mathematics and Computer Science
Babes-Bolyai University of Cluj
3400 Cluj, Romania

*Corresponding author; email: tgrosan@math.ubbcluj.ro

Received: 24 November 2010
Accepted: 21 May 2011